

Wellcome back

# CSC872 Pattern Analysis and Machine Intelligence

Spring 2025

Department of Computer Science  
San Francisco State University

CSC872: PAMI – Kazunori Okada (C) 2025

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## Course Information

- **URL:** <https://bidal.sfsu.edu/~kazokada/csc872/>

- **Instructor**

- **Kaz Okada**, kazokada@sfsu.edu
- OH: TH911, Thr: 11:30am – 12:30pm



- **TA:**

- **Shivani Bokka**, sbokka@sfsu.edu
- OH: ~~Zoom~~, Mon: 2:00pm – 3:00pm



- See the *Canvas* page for other information.
- Policies: Please read the course website above.
- Be aware of the deadline and the late policy etc!!!

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## Basics

Hello! (y)

- Course slides shared online **after** each lecture through the course website: Click links in the “note” column of the “lecture plan” table.
- Attendance are taken in-person.
- Honor Code:
  - I request all students to read and pledge on the course honor code (found in Canvas: general information).
  - No assistive sites (chegg, numerade) No ChatGPT etc
  - Read it thoroughly and send your pledge on Canvas **by Feb 4<sup>th</sup>**. See instructions given in the Canvas link.

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## Waitlisted

- For those who were waitlisted, we still have to see if we can accommodate you. Please stay tuned and keep attending lectures and do assignments.
- If I choose to add you with permit number after the 2<sup>nd</sup> week, automatic email should be sent to you with the permit number that you can use to add to this course. Hope to resolve this soon.

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## Evaluations

- No Midterm/Final Exams
- Homework (50% of total grades)
  - Five HWs (See course website for schedule)
  - **Shared from/Submitted to Canvas links**
  - Due in one/two week(s)
  - Includes some difficult analytical problem solving
- Final Project (25% report, 10% present)
  - Final Report (**Literature Survey Report**) due on May 13.
  - Final Presentation (Presentation) on May 13.
  - Assignments given in the course website (follow the link)
- Fast Prototyping (15%: 5% each for 3 exers)

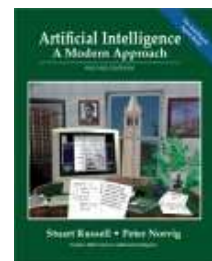
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## Text Books

- **AIMA by Russell-Norvig: our text, general AI**
  - Duda-Hurt: for PR foundation
  - Hastie: advanced ML
  - Gonzales-Woods: comprehensive IP&CV
  - Bishop: PR and NN



- **Read the AIMA chapters before the classes**
- Additional reading assignments given as appropriate

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## Course Schedule

- Consult the course website for details of the **lecture plan**
- First part: lecture 4:00 – 5:30 ca (90min)
- 10 min break, for exercise and restroom
- Next part: in-class exercise 5:40 – 6:45 (65min)
- Drop deadline: Feb 17 (3 weeks)

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## Course Overview

- ~~Intro & Agent~~ (AI: Ch1-2) *Classic AI*
- ~~Search Methods~~ (AI: Ch3-4)
- ~~Logic and Inference~~ (AI: Ch7-9) *Pattern Recognition*
- ~~Bayesian Framework~~ (PR: Ch13-14 etc)
- ~~Statistical Modeling~~ (PR: Ch20 etc)
- ~~Statistical Classification~~ (PR: Ch3 etc)
- ~~Machine Learning~~ (ML: Ch18 etc)
- ~~Supervised Classification~~ (ML: Ch3 etc)
- ~~Supervised Regression~~ (ML: Ch3 etc)
- ~~Function Learning~~ (NN: Ch20 etc)
- ~~Deep Learning~~ (NN)

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## MATLAB

- You will learn how to use a powerful prototyping software environment !!!
  - Exercise Tutorials
  - Fast Prototyping
- You need to bring a laptop with MATLAB **by the next lecture**
  - Free copy available for all SFSU students:  
<https://athelp.sfsu.edu/hc/en-us/articles/360011475074-Getting-MATLAB-for-students> (Go to Canvas:Matlab)
  - Follow instruction to install and make sure it can start on your laptop
  - Play a little before next lecture

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## In-Class Exercises

- MATLAB Exercise
  - Basics of MATLAB
  - Three exercise-sessions
  - Hands-on tutorials
  - TA and my help during office hours
  - **End up learning a useful tool**
- Fast Prototyping Exercises
  - Hands-on MATLAB software prototyping guided exercise
  - Three Exercises
    - 3 classic algorithms: PCA, Mean Shift, LDA
    - 3 computer vision problems
  - 3 sessions for each exercise
  - 15% of the total grades!
  - **End up learning how to quickly implement your ideas defying all the nice thing you learned in SE classes.**
- **Bring your own laptop with MATLAB!!!**

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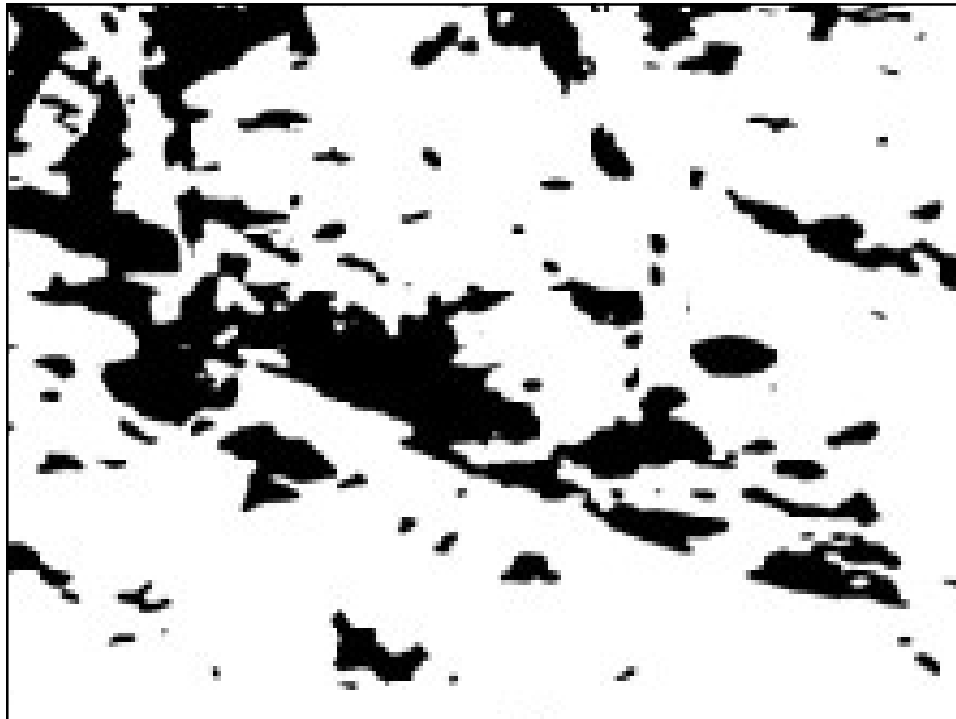
# Introduction: the PAMI framework

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Pattern Analysis and Machine Intelligence

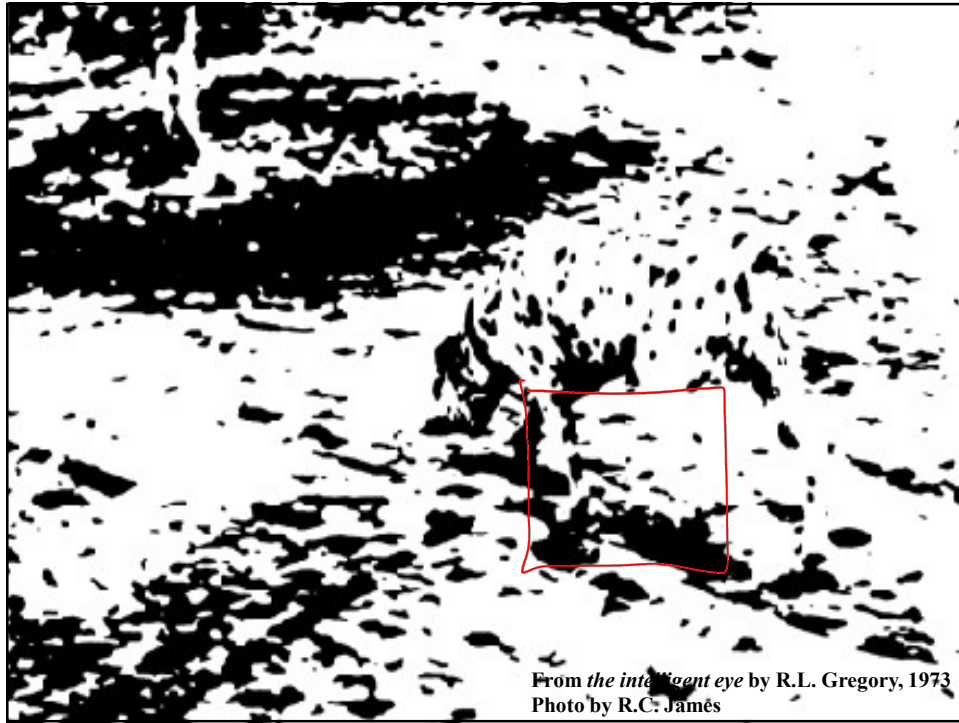
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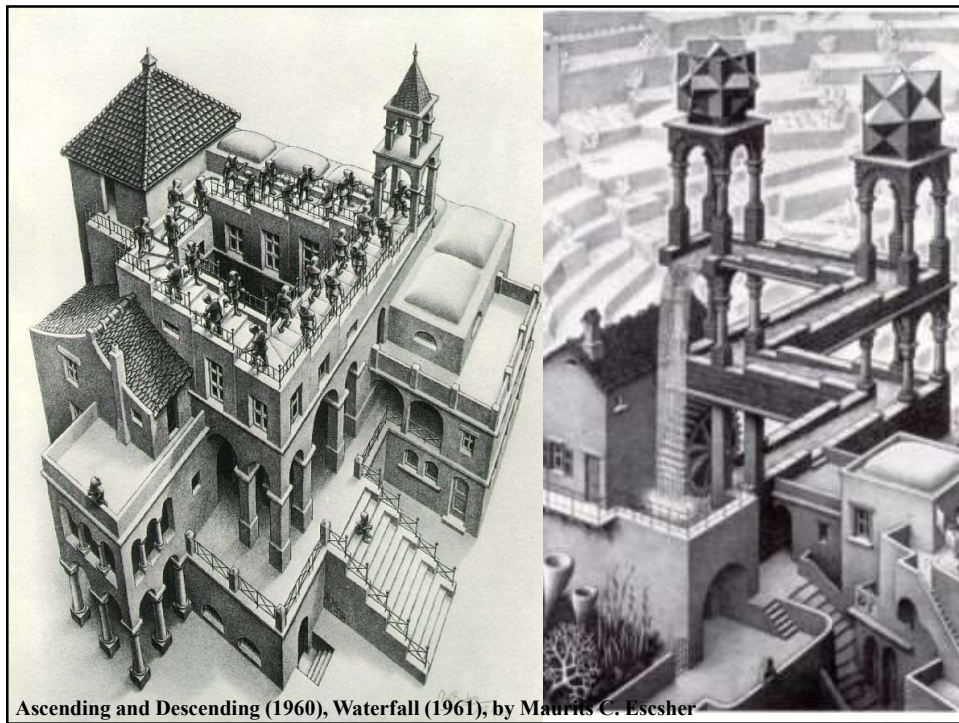
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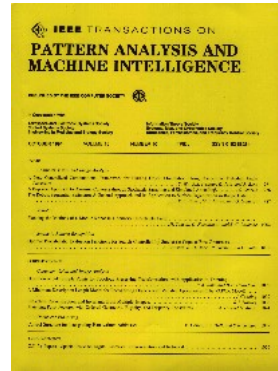
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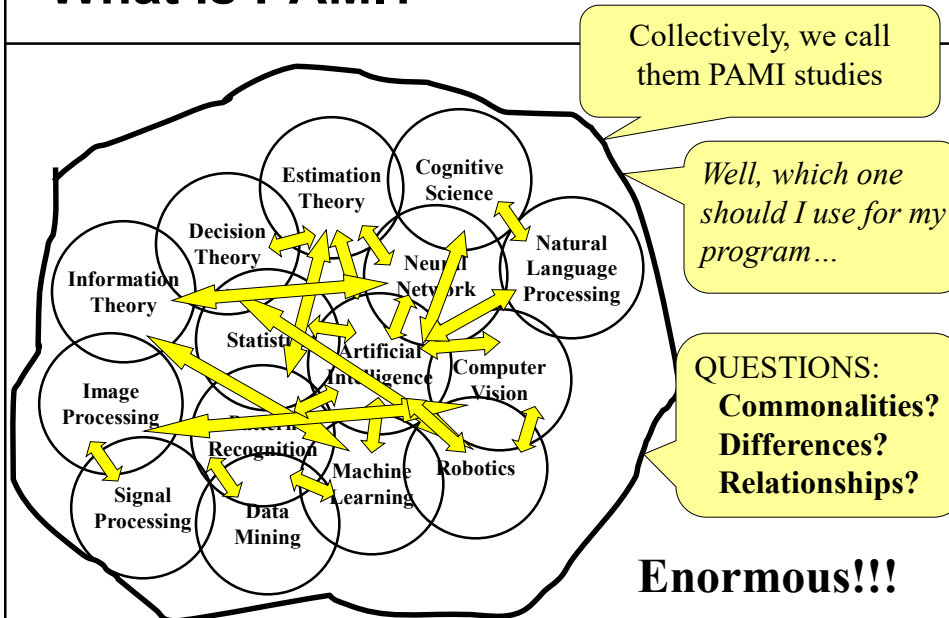
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# What is PAMI?

- Pattern Analysis and Machine Intelligence is a study for:
  - A classic artificial intelligence
  - Understanding the foundation of different approaches to make machines behave intelligently
  - Applying AI techniques to various engineering tasks
  - Type of researches that get published in IEEE trans on PAMI ...

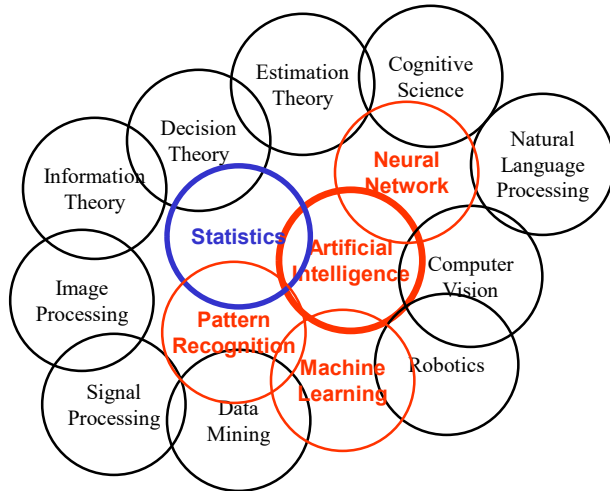


# What is PAMI?





# What is PAMI?



**Focus on**  
**AI, PR, ML, NN**  
**& Statistics**

**Emphasizing**  
 - the three questions  
 - quantitative statistical approach

**Upon further interests, you should study each subject further**

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# The 3 questions: Common Framework

- What is PATTERN?
    - Codifying Properties of World
    - **Data & Knowledge Representation** ?
  - What is MACHINE INTELLIGENCE?
    - Formalizing Intelligence for Machines
    - **Problem Formulation** ?
  - What is ANALYSIS?
    - Analyzing Data & Knowledge to solve formulated prob.
    - **Problem Solving** ?
- Common Questions**
- For Different Approaches Different Answers you get**

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## Data & Knowledge Representation

- How to formally describe data/knowledge?
- **Algebraic Variables**
  - Boolean, Scalar, Vector, Matrix, Tensor
- **Probabilistic Variables and Distributions**
  - Random Variables, Probabilistic Mass/Density Function
- **Formal Rules**
  - Rational Statement, Causality
- **Discrete & Continuous Relations**
  - Tree, Graph, Function, Ontology

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## Problem Formulation: Problems?

- **Problems: what is the computational task?**
  - Inference
  - Modeling
  - Learning
  - Classification
  - Regression

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## Problem Formulation: Formulations?

- **Formalisms: How to describe the task?**
  - Agents
  - First Order Logic
  - Bayesian Inference/Classification
  - Maximum Likelihood Estimation (MLE)
  - Maximum A Posteriori Estimation (MAP)
  - Statistical Regression
  - Energy/Error Minimization
  - Maximum Information
  - Ensemble Learning

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## Problem Solving: Basics

- **How to solve the problem w/ given data?**
  - Search: Depth-First, Width-First, A\*
  - Logical Inference: Resolution
  - Kernel Density Estimation (KDE)
  - Expectation-Maximization (EM) Algorithm
  - Principal Component Analysis (PCA)
  - Linear Discriminant Analysis (LDA)
  - Hill-Climbing/Gradient Descent
  - Simulated Annealing
  - Back Propagation
  - Support Vector Machine (SVM)
  - Markov Chain Monte Carlo (MCMC)
  - AdaBoost, Random Forest, XGBoost, CNN, RNN...

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## PAMI Framework

- **KR** = Data & Knowledge Representation
- **PF** = Problem Formulation
- **PS** = Problem Solving
- **Make your habit to think everything in the form of (KR-PF-PS)**
- Example: **you** as a PAMI problem...
  - KR: your brain with all the details therein
  - PF: maximize amount and quality of learning
  - PS: taking and working in this course

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## Review: Basic Concepts

- Some relevant mathematical ideas:
  - Calculus (high-school to lower-division)
  - Algebra (high-school to lower-division)
  - Probability (basic + some advanced)
  - Statistics (basic + some advanced)
- You want to make sure you are comfortable with these concepts and notations
- OK... some refresher now;

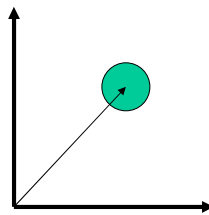
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## KR: Variable: Scalar & Vector

- **Variable** is:
  - Symbolic representation of quantity
  - Unknown quantity that can change in algebraic sense
  - Measurable attribute of a system in statistics
- **Scalar X** : Variable indicating a single-valued entity
- **Vector X** : Variable indicating a multiple-valued entity



$$\begin{pmatrix} x \\ y \end{pmatrix}^T = (x, y)$$
$$x = (x \ y)^T = \begin{pmatrix} x \\ y \end{pmatrix}$$

$x = a$  : area  
 $x = \mu$  : angle  
**Dimension** := number of coeffs

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## KR: Continuous vs. Discrete

- **Continuous Variable X**

- indicates real-value entities
- $x \in \mathbb{R}$
- $x = (x_1, \dots, x_n)^T \in \mathbb{R}^n$

$$\overbrace{\mathbb{R} \times \mathbb{R} \times \mathbb{R} \times \dots \times \mathbb{R}}^n$$

N-Dim vector

- **Discrete Variable X<sub>i</sub>**

- only take a set of predetermined discrete values
- $x_i \in \mathbb{N}$       Natural number:  $i = 1, 2, \dots$
- $x_i \in \{MO, TU, WE, TH, FR, SU, SA\}$

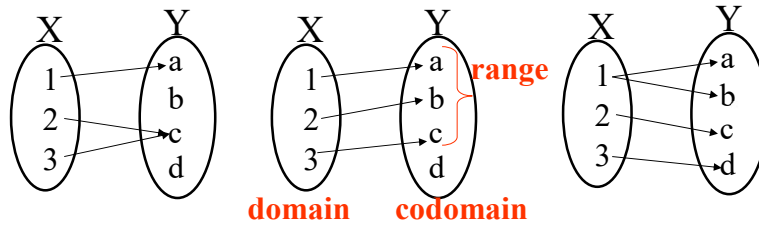
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## KR: Function

- Deterministic dependence of two quantities/sets, associating input  $X$  to output  $Y$  by a binary relation



- $f: X \rightarrow Y$
- Map, Mapping, Transformation = Function
- Inverse function:  $g = f^{-1}: Y \rightarrow X$

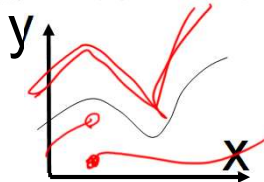
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## KR: Function Properties

- Rules of  $f$  described in a graphical plot or sometimes in analytic formula when known



$$y = f(x)$$
$$x \in X$$
$$y \in Y$$

- Continuous Function
- Differentiability
- Smooth Function
  - All-order differentiable over entire domain

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# KR: Matrix

**m-by-n matrix**

$$A = \begin{matrix} & \begin{matrix} \text{n columns} \end{matrix} & \xrightarrow{\text{j changes}} \\ \begin{matrix} a_{i,j} \\ \text{m rows} \\ \downarrow \text{i changes} \end{matrix} & \begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} & \dots \\ a_{2,1} & a_{2,2} & a_{2,3} & \dots \\ a_{3,1} & a_{3,2} & a_{3,3} & \dots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} \end{matrix}$$

## Vector as a Matrix

$$x = \begin{pmatrix} x \\ y \end{pmatrix} = (x \ y)^T$$

$$x^T = (x \ y)$$

- Product does not commute:  $AB \neq BA$
- Transpose:  $A^T : a_{ij} \leftarrow a_{ji}$ , (swapping rows & columns):  $(AB)^T = B^T A^T$
- Symmetric matrix  $A$ :  $A^T = A : a_{ij} = a_{ji}$
- Inverse matrix of  $A^{-1} : A A^{-1} = A^{-1} A = I_n$
- Orthogonal matrix  $A$ :  $A^T = A^{-1} ; AA^T = A^T A = I_n$
- Outer and inner product

$$xx^T = \begin{pmatrix} x^2 & xy \\ xy & y^2 \end{pmatrix}$$

$$x^T x = x^2 + y^2 = \text{tr}(xx^T)$$

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# PF: Matrix Equations

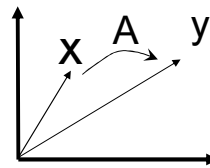
- Linear equations

$$\begin{matrix} ax_1 + bx_2 = e \\ cx_1 + dx_2 = f \end{matrix} \Rightarrow Ax = b; A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}; b = \begin{pmatrix} e \\ f \end{pmatrix}$$

$$x = A^{-1}b$$

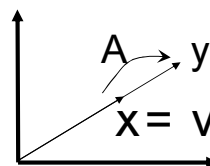
- Linear transform

$$\begin{matrix} y = Ax \\ x = A^{-1}y \end{matrix} \quad A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \Rightarrow$$



- Eigen-values & vectors

$$\lambda v = Av; \lambda \in R$$



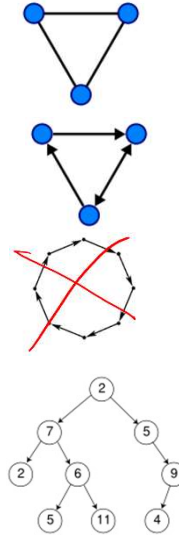
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## KR: Graph

- Undirected graph:  $G=(V,E)$ 
  - vertices and edges (no direction)
- Directed graph:  $G=(V,A)$ 
  - vertices and arrows
- Directed acyclic graph (DAG)
  - Directed graph without a loop
- Connected graph
  - Can reach from any vertex from any other vertices
- Connected DAG
  - **Tree**



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## KR: Boolean Variable

- **A** is a Boolean variable if it indicates two-valued system, a statement or event
  - e.g., indicator variable  $A = \{\text{Yes, No}\}$
  - e.g.,  $A = \text{My name is George}$
  - e.g.,  $A = \text{I teach CSC872}$
- Some event has intrinsic degree of **uncertainty** as to whether **A** occurs
  - e.g.,  $A = \text{There will be an earthquake tomorrow}$
  - e.g.,  $A = \text{My stock price will go up tomorrow}$
- **Random Variable** is a function that chooses a value from the event space  $\{\text{True, False}\}$  according to probability  $P(A)$

$$P(A) = 0.0001$$

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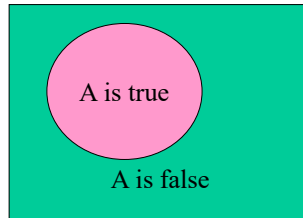


# KR: Basic Probability

- $P(A)$  means “the fraction of possible worlds in which  $A$  is true”

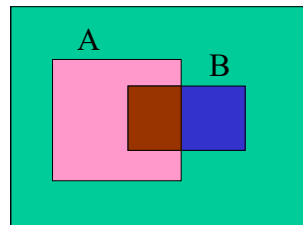
Event space of  
all possible  
worlds

Its area is 1



$P(A)$  is the area  
of the pink circle

- The axioms of probability !!!
  - $0 \leq P(A) \leq 1$
  - $P(\text{True}) = 1$
  - $P(\text{False}) = 0$
  - $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$



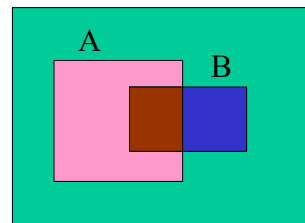
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# KR: Probability Cont.

- Joint Probability
  - Probability of two events in conjunction
  - $P(A \text{ and } B) := P(A \wedge B) := P(A, B)$



- Marginal Probability
  - Probability of one event ( $A$ ) regardless of the other events ( $B$ )
  - Obtained by summing (integrating) a joint probability over the event space  $\Omega$  for unwanted events ( $B$ )

$$\begin{aligned}
 - P(A) &= \sum_{v \in \Omega} P(A \wedge B=v) \\
 &= P(A \wedge B) + P(A \wedge \sim B)
 \end{aligned}$$

$$- P(\text{not } A) = P(\sim A)$$

$$- P(A) + P(\sim A) = 1 \quad \text{total probability theorem}$$

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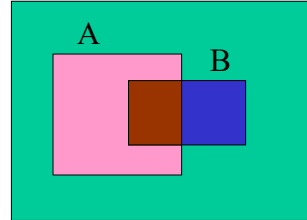
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# KR: Probability Cont.

- **Conditional Probability**

- Probability of an event (A) given other event (B)
- $P(A|B) = \frac{\text{area of } A \text{ and } B}{\text{area of } B}$
- $P(A|B) = \frac{P(A \wedge B)}{P(B)}$
- $P(\text{Yes}|\text{Math}) ? = \frac{P(\text{Yes}, \text{Math})}{P(\text{Math})} = \frac{2/8}{4/8} = \frac{2}{4} = \frac{1}{2}$



- **Product Rule**

- Joint probability can be written as a product of a conditional and a marginal
- $P(A \wedge B) = P(A|B)P(B) = P(B|A)P(A)$

- **Statistical Independence satisfies**

- $P(A \wedge B) = P(A)P(B)$
- $P(A|B) = P(A)$
- $P(B|A) = P(B)$

X = College Major  
Y = Likes "XBOX"

X	Y
Math	Yes
History	No
CS	Yes
Math	No
Math	No
CS	Yes
History	No
Math	Yes

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# KR: Beyond Boolean Events

- When more than one state (over a discrete variable):

- e.g., X = day,  $\Omega := \{\text{Mon}, \dots, \text{Sun}\}$
- **Discrete Random Variable**

- $P(X = v_i \text{ and } X = v_j) = 0 \text{ if } i \neq j$

(mutually exclusive)

- $P(X = v_1, \dots, X = v_k) = \sum_{i=1}^k P(X = v_i) = 1$

(total prob. Th.)

- $P(Y) = \sum_{i=1}^k P(Y \text{ and } X = v_i)$

(marginal)

- When over continuous variable:

- **Continuous Random Variable**
- e.g., X = temperature of SF

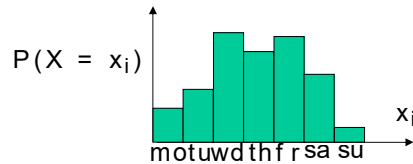
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## KR: Probability Distribution

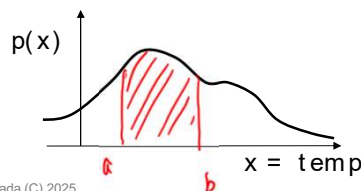
- For a discrete random variable  $X$ 
  - Probability Mass Function



$$\sum_{i=1}^k P(X = x_i) = 1$$

total probability theorem

- For a real-valued random variable  $X$ 
  - Probability Density Function



$$\int_{-\infty}^{+\infty} p(x) dx = 1$$

$$P(a \leq X \leq b) = \int_a^b p(x) dx$$

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## KR: Expectation

- For a discrete random variable  $X$ 
  - $E[X] = \sum_{\Omega} x_i P(X=x_i) = \mu$  (population mean)
  - $E[f(X)] = \sum_{\Omega} f(X=x_i) P(X=x_i)$

- For a real-valued random variable  $X$ 
  - $E[X] = \int_{\Omega} x P(y) dy$

$$E[X] = \mu_x \quad E[Y] = \mu_y$$

- Linearity

$$E[aX+Y] = aE[X]+E[Y] \approx a\mu_x + \mu_y$$

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## PF: Statistics

- Independent and Identically-Distributed (i.i.d.) Random Variable
  - Rolling a fair dice for instance.
  - If  $x_1, x_2, x_3, \dots, x_i, \dots, x_k$  are i.i.d. of  $X$  then
  - $P(x_1, x_2, x_3, \dots, x_i, \dots, x_k) = P(X=x_1)P(X=x_2)\dots P(X=x_k)$
- Central limit theorem
  - The sum of i.i.d. random variables with finite variance will be approximately normally (Gaussian) distributed as we go towards an infinite number of samples.
  - A reason why you see a lot of Gaussians ...