Prototype Exercise 3

Exercises 11, 12, 13
CSC872
Pattern Analysis and Machine Intelligence

https://bidal.sfsu.edu/~kazokada/csc872/Face Classification_Data.zip

Fast Prototyping Exercise

• Fast Prototyping
  – Learn how to do a quick proof of concept by building a prototype
  – Correctness matters (no sloppy algorithm!)
  – Speed matters (no beautification!)
  – No perfect SE necessary
  – No copying of codes online.
  – Parameterization/Visualization/Experimentation
    – Find out what are free parameters in your algorithm whose value must be hand-picked by you
    – Learn how to view internal variable’s current values
    – Learn how to visualize your prototype’s results in plots/images etc
    – Tweak the parameter values and study your prototype’s behavior to understand the how algorithm works

• Group Work
  – You are encouraged to freely exchange ideas and codes
  – Contributions to others are as valuable as making your own work
Fast Prototyping Exercise

• Please upload your matlab codes thru iLearn forum for my grading and your playing!
  – First two exercises: Due on midnight of the day (just what you did during the exercise)
  – Third last exercise: Due on noon next day (complete version with some doc/screen shots of running the code)

• 5% extra credit for those who completed all three algorithms in-class
  – Those who finished during the 3 exercises received a note from me in iLearn.
  – Please remind me if you had finished during the exercises but did not receive my note of your successful completion in ilearn.

Platforms

• MATLAB
  – MathWorks: http://www.mathworks.com/

• MATLAB clones
  – Octave: http://www.gnu.org/software/octave/
  – SciLab: http://www.scilab.org/
Public Libraries

- OpenCV (Computer Vision)

- ITK (Medical Imaging)
  - http://www.itk.org/

- WEKA (Machine Learning)

Classification of Facial Gender

- Learn an LDA classifier that classify input image to either female or male (binary classification)
- Smaller sized images are provided
### Paper 3

- **Swets and Weng**  
- **Using Discriminant Eigenfeatures for Image Retrieval**  
- **PAMI, 18(8): (1996)**

- A.M. Martinez, A.C. Kak,  
- [http://www.ece.osu.edu/~aleix/](http://www.ece.osu.edu/~aleix/)

### Data

- The same but reorganized data from the FP#1 for face recognition  
- [https://bidal.sfsu.edu/~kazokada/csc872/FaceClassification_Data.zip](https://bidal.sfsu.edu/~kazokada/csc872/FaceClassification_Data.zip)

- Images are organized in 2 folders  
- Female: 54 32x32 8bit facial images  
- Male: 45 32x32 8bit facial images
Linear Discriminant Analysis

- Supervised learning for classification
- Input: Grayscale Image
  - 8bit (0-255) grayscale images of 32x32 size
- LDA projection: linear transformation to a 1D space
- Threshold-based Classification: find the Bayes optimal threshold for the data after projection.
- Output: binary class labels
  - E.g., female (+1), male (-1)

LDA Setting

- Feature extraction (projection): \( y = w^T x \)
- LDA: Find \( w \) such that within-class scatter (\( S_w \)) is minimized and between-class scatter (\( S_b \)) is maximized.
- LDA solution is given by solving a generalized eigen-value problem
  \[ S_b w = \lambda S_w w \]
  \[ [V D] = \text{eig}(S_b, S_w) \]
LDA Limitations

• You get only $K-1$ non-zero eigen-vectors where $K$ is the # of classes

• You need at least $K+d$ samples to have non-singular $S_w$ where $d$ is the dimensionality of inputs
  – Singular matrix cannot be inverted!!!

PCA+LDA Solution

• Because $S_w$ is singular you cannot solve the generalized eigenvalue problem

• Soln:
  – First Perform PCA on the entire data set
  – Find a subset with $K$ top PCs that capture most of data variance
  – Project all the input data points to the PCA subspace
  – Compute the $S_w$ and $S_b$ with the projected datapoints in the PCA space (low-dimensional space)
  – Perform LDA
  – Compute the slope and intercept of the discriminant function from the LDA results
Formulae

\[ X = \{x_1, x_2, \ldots, x_k, \ldots, x_K\} \]
\[ M_k = |X_k|: \text{ the number of samples in } k\text{–th class} \]
\[ \mu_k = \frac{1}{M_k} \sum_{i=1}^{M_k} x_i: \text{ mean of } k\text{–th class} \]
\[ \mu = \frac{1}{|X|} \sum_{k=1}^{K} M_k \mu_k: \text{ mean of } X \]
\[ S_w = \sum_{k=1}^{K} \sum_{i=1}^{M_k} (x_i - \mu_k)(x_i - \mu_k)^t \]
\[ S_b = \sum_{k=1}^{K} M_k (\mu_k - \mu)(\mu_k - \mu)^t \]

LDA: find \( w \) that maximizes \[ J(w) = \frac{w^t S_b w}{w^t S_w w} \]

Formulae

LDA: find \( w \) that maximizes \[ J(w) = \frac{w^t S_b w}{w^t S_w w} \]

\[ S_b w_m = \lambda_m S_w w_m \]
\[ (S_w^{-1} S_b) w_m = \lambda_m w_m \]
Algorithm

1) Given labeled data $X = \{X_p, X_n\}$: female ($p$), male ($n$)
2) Do PCA  
   \[ CV = VD \]
   \[ V = \{PC_1, ..., PC_N\} \]
   \[ D = \text{diag}(ev_1, ..., ev_N) \]
3) Find top $K$ PCs that cover $9X\%$ of the variance
4) Form PCA model $F$  
   \[ F = \{PC_1, ..., PC_K\}^T \]
5) Project all data points to $FX = \{P_p, P_n\}$:
6) Compute $S_w$ and $S_b$ from $\{P_p, P_n\}$  
   \[ p_i = F \cdot (x_i - \mu) \]
7) Solve a generalized eigen-value problem with $S_w$ and $S_b$
8) This results in a single PC: $v$
9) Compute discriminant slope $w = v^\prime \cdot F$
10) Compute discriminant intercept $b = v^\prime \cdot (meanP_p + meanP_n)/2$
11) The result if $w^\prime (x-\mu) - b$ is positive then $+1$ otherwise $-1$, (check if the sign is right)

Useful MATLAB Codes

For LDA
- Set $X$ as a matrix with each row is a vectorized face
- $m = \text{mean}(X)$: sample mean of $X$
- $S = \text{cov}(X)$: covariance matrix (mean removed)
- Scatter matrix = $\text{cov}(X) \cdot (N-1)$  
  $N$: # of samples in $X$
- $[V D]=\text{eig}(A, B)$: generalized eigenvalue problem solver
- hist (projected $X1$ and $X2$): create a histogram