Overview

- Last lecture
  - Medical Image Segmentation II
  - Level Set Method
  - Performance Validation
- Today's lecture
  - Medical Image Registration I
  - Overview of Registration
  - Rigid Registrations
  - Landmark-based: Minimizing Fiducial Registration Error
  - Surface-based: Iterative Closest Points

Medical Image Registration I:
Rigid Cases

CSC621-821
Biomedical Imaging and Analysis
Dr. Kazunori Okada
Review: Medical Image Modalities

- **Anatomical**
  - CT, MRI, X-ray
  - Depicting primarily morphology

- **Functional**
  - PET, SPECT, fMRI
  - Depicting primarily information on the metabolism of the underlying anatomy

Medical Image Integration

- **Registration**
  
  Bring the multiple images into spatial alignment

- **Fusion**
  
  Integrated display of the data involved
  
  Matching, Co-registration, Correlation,...
What is Registration?

- Aligning two images (source and target)
- Matching two images so that corresponding coordinate points in the two images correspond to the same physical region of the scene being imaged
- Mathematically, determining the transform that maps points in the target image to points in the source image

Point Correspondence in 3D

Coordinate Transform: $T$

$$(y_1, x_1) \rightarrow (y'_1, x'_1)$$
Application 1: Multimodal Matching

- Matching images of the same patient taken by different sensors (inter-modal)
  - Example: MRI + SPECT, CT + PET
  - Generating composite images (Fusion)
  - ‘Big picture’ view
  - Study how different features interact and combine

Combining MRI and CT
Combining MRI and SPECT

Application 2: Temporal Matching

- Matching two images of the same patient with the same instrument but at different times (intra-modal)
  - Example: CT + CT or MRI + MRI
  - Examine patient recovery
  - Analyze effectiveness of treatments (therapy monitoring)
  - Early detection of medical problems (change analysis)
Therapy Monitoring

Functional MRI Example
More Applications: What for?

- **Diagnosis**
  - Combining information from multiple imaging modalities
  - Detect tumors and disease lesions
- **Studying disease progression**
  - Monitoring changes in size, shape, position or image intensity over time
- **Image guided surgery or radiotherapy**
  - Relating pre-operative images and surgical plans to the physical reality of the patient
- **Patient comparison or atlas construction**
  - Relating one individual’s anatomy to a standardized atlas
- **Biomedical Research**
  - Classifying blood cells and chromosomes

Visualization of Registered Images

- **Mosaicing CT & MRI**

![Mosaicing Images](image-url)
Basic Concept

A: Fixed/Reference

B: Moving/Target

Are they similar?

NO

but better

OK Aligned!!!
Registration: Terminology

- 2 input images
  - Fixed (reference/target) and moving (test/source)

- Cost Function (Metric)
  - determines the “fitness/similarity/distance” of the current registration iteration

- Optimizer (Update)
  - adjusts the transform in an attempt to improve the metric (compute a new transform)

- Interpolator (Transform)
  - applies transform to image and computes sub-pixel values (compute a new image)

Registration Framework

Cost function calculation dominates for 3D images and is inherently parallel
Transformation Types

- Relates the position of features in two images
  - **Rigid**
    - Translations and rotations
  - **Affine**
    - Also allows scaling and shearing/skewing
  - **Perspective/Projective**
    - The parallelism of lines need not be preserved
  - **Curved**
    - Allows the mapping of straight lines to curves
  - **Deformable**
    - Free-form mapping

Deformable Transformation

- **Curved**
- **Free-form**
**Dimensionality**

- 3D/3D registration of two images
  - Your projects
- 2D/2D registration
  - Less complex by an order of magnitude where both the number of parameters and the volume of the data are concerned.
- 2D/3D registration
  - Direct alignment of 3D spatial data (CT) to 2D projective data (x-ray)
  - The alignment of a single tomographic slice (Ultrasound) to 3D spatial data (MRI)
- 2D examples for 3D cases...

**Rigid Transformation**

Distances between all points remain constant.

**Rigid**

- 3 degrees of freedom

**Non-rigid**

- Not 3 degrees of freedom
Linear Transformation

- A mapping $T$ of $\mathbb{R}^n$ into $\mathbb{R}^m$, written as
  $$ T: \mathbb{R}^n \rightarrow \mathbb{R}^m $$
  is a rule that assigns to each vector $u$ in $\mathbb{R}^n$ an unique vector $v$ in $\mathbb{R}^m$
- Format: $T(x,y,..|a,b...)$:
  - $(x,y,..)$ spatial variables
  - $(a,b,..)$ free parameters for transformation
- Linearity: must be operation preserving
  $$ T(u + v) = T(u) + T(v) $$
  $$ T(cu) = cT(u) $$

2D Rotation and Translation

- Rotate point $(x,y)$ counterclockwise at angle $\theta$
  $$ \begin{pmatrix} x' \\ y' \end{pmatrix} = T(x, y | \theta) = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x' \\ y' \end{pmatrix} $$
  DOF = 1
- Translation in the $xy$ plane where $a,b \in \mathbb{R}$
  $$ \begin{pmatrix} x' \\ y' \end{pmatrix} = T(x, y | a,b) = \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} x+a \\ y+b \end{pmatrix} $$
  DOF = 2
2D Rigid Transformation

- Rotate point \( x=(x,y) \) counterclockwise at angle \( \theta \) and translate by \( T=(a,b) \)

\[
T(x \mid \theta \in \mathbb{Z}, a \in \mathbb{R}, b \in \mathbb{R})
\]

\[
y = T(x \mid \theta, a, b) = Rx + T
\]

\[
= \begin{pmatrix}
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta
\end{pmatrix}
\begin{pmatrix}
x \\
y
\end{pmatrix}
+ \begin{pmatrix}
a \\
b
\end{pmatrix}
\]

\[
\begin{pmatrix}
x' \\
y'
\end{pmatrix} = \begin{pmatrix}
x \cos \theta - y \sin \theta + a \\
x \sin \theta + y \cos \theta + b
\end{pmatrix}
\]

3D Rotation

\[
R_{xyz}(\theta_1, \theta_2, \theta_3) = R^{(1)}_{yz}(\theta_1) \ast R^{(2)}_{xz}(\theta_2) \ast R^{(3)}_{xy}(\theta_3)
\]

\( R^{(i)} \) rotates an image along axis \( i \) by angle \( \theta_i \)

- \( R^{(1)}_{yz}(\theta_1) = \begin{pmatrix}
1 & 0 & 0 \\
0 & \cos \theta_1 & -\sin \theta_1 \\
0 & \sin \theta_1 & \cos \theta_1
\end{pmatrix} \)

- \( R^{(2)}_{xz}(\theta_2) = \begin{pmatrix}
\cos \theta_2 & 0 & \sin \theta_2 \\
0 & 1 & 0 \\
-\sin \theta_2 & 0 & \cos \theta_2
\end{pmatrix} \)

- \( R^{(3)}_{xy}(\theta_3) = \begin{pmatrix}
\cos \theta_3 & -\sin \theta_3 & 0 \\
\sin \theta_3 & \cos \theta_3 & 0 \\
0 & 0 & 1
\end{pmatrix} \)
3D Rigid Transformation

- Rotate point \( x = (x, y, z) \) counterclockwise at angle \( \theta = (\theta_1, \theta_2, \theta_3) \) and translate by \( T = (a, b, c) \)

\[
y = T(x | \theta_1, \theta_2, \theta_3, a, b, c) = R^1 R^2 R^3 x + T
\]

6 degree of freedom

\[
\begin{pmatrix}
  x' \\
y' \\
z'
\end{pmatrix} = R^1_{yz} (\theta_1) R^2_{xz} (\theta_2) R^3_{xy} (\theta_3)
\begin{pmatrix}
x \\
y \\
z
\end{pmatrix} + \begin{pmatrix}
a \\
b \\
c
\end{pmatrix}
\]

Can be pre-multiply to a 3x3\( R_{xyz} \)

In Homogeneous Coordinates

- Simplifying the transform into a single matrix!
- \( y = Rx + T \rightarrow y = Ax \) (Why?)
- How? Ans: Add one more dimension to \( x \):

\[
(x_1, x_2, x_3) \rightarrow (x_1, x_2, x_3, 1)
\]

\[
\begin{pmatrix}
y_1 \\
y_2 \\
y_3 \\
1
\end{pmatrix} = \begin{pmatrix}
3 & 0 & 0 & x_1 \\
0 & 3 & 0 & x_2 \\
0 & 0 & 3 & x_3 \\
0 & 0 & 0 & 1
\end{pmatrix}
\]

\[
\begin{pmatrix}
\tilde{x}_1 \\
\tilde{x}_2 \\
\tilde{x}_3 \\
1
\end{pmatrix} = \begin{pmatrix}
\tilde{x}_1 \\
\tilde{x}_2 \\
\tilde{x}_3 \\
1
\end{pmatrix}
\]

\[
\tilde{x} = \begin{pmatrix}
x_1 \\
x_2 \\
x_3 \\
1
\end{pmatrix}
\]

\[
\tilde{y} = \begin{pmatrix}
y_1 \\
y_2 \\
y_3 \\
1
\end{pmatrix}
\]
Various Transforms in Matrix Form

- **Rigid (Similarity) Transform**
  - $r$: 3x3 rotation matrix
  - $t$: 3D translation vector

- **Scaling**
  - Replace $1$ by a positive value $S$

- **Affine Transform**
  - $r$ becomes arbitrary real-valued matrix

$$
\begin{pmatrix}
  y_1 \\
  y_2 \\
  y_3 \\
  1
\end{pmatrix}
= \begin{pmatrix}
  r & t \\
  0 & 1 \\
\end{pmatrix}
\begin{pmatrix}
  x_1 \\
  x_2 \\
  x_3 \\
  1
\end{pmatrix}
$$

Spatial Transform + Image Interpolation

Transform $T(x|\theta)$

Interpolate (bicubic: lec4)
How to Update Transforms?

- **Optimization!**
- **Goal:**
  - Registering moving image \( M \) to fixed image \( F \) using \( T(\theta) \)
- **Process:**
  - Iteratively find the best transform \( T \)'s parameter \( \theta \) that maximizes cost (similarity metric) \( C(F,M) \) or minimizes distance between \( F \) & \( M \) \( C(F,M) \)
- **So the results depend on**
  - What Cost/Metric you use
  - What Transform you use
  - What strategy you use to maximize/minimize \( C(F,M) \)

Two Types of Optimization

- **Two Basic Types**
- **Direct computation**
  - \( \theta(t+1) = \theta(t) + \Delta \theta(t) \)
  - **Explicit formula for the delta** as a function of data
  - Gradient ascent algorithm
- **Neighborhood Search**
  - **Greedy algorithm** with a neighborhood search
  - Look for the best change in the neighborhood of the current parameter value
**Formula for Gradient Ascent**

- Derivative of similarity metric $C$ with respect to the unknown parameter $\theta$

$$\Delta \theta = \frac{\partial C(\theta | F, M)}{\partial \theta} = J \cdot \Delta \theta$$

$$y = \mathbf{R}_{\mathbf{x}} \mathbf{1}$$

$$J_{20R} = \begin{bmatrix} \frac{2y_1}{x_1} & \frac{2y_1}{x_1} & \frac{2y_1}{x_1} \\ \frac{2y_2}{x_2} & \frac{2y_2}{x_2} & \frac{2y_2}{x_2} \\ \vdots & \vdots & \vdots \\ \frac{2y_m}{x_m} & \frac{2y_m}{x_m} & \frac{2y_m}{x_m} \end{bmatrix}$$

**More Advanced Algorithms**

- **Derivative-based**
  - Powell’s method
  - Downhill simplex method
  - Levenberg-Marquardt optimization

- **Stochastic**
  - Simulated annealing
  - Genetic methods

- **Search-based**
  - Quasi-exhaustive searching
Registration Algorithm Types

- **Point-based (Landmark-based)**
  - Landmarks / feature points selected by the user or algorithm or physical markers
  - Only points / geometrical information used

- **Surface-based (Segmentation-based)**
  - Align binary surface structures
  - Pre-segmentation is needed

- **Intensity-based (Voxel-based)**
  - Minimize intensity difference over entire image
  - Intensity information drives the registration
  - Geometrical information can be combined
Metric Overview

- Scalar function $C(\theta, F, M, T)$ of the transform parameters $\theta$ for a given fixed image $F$, moving image $M$, and transformation type $T$

- Fiducial Registration Error (FRE)
- Mean Squares Distance (MSD)
- Normalized Cross Correlation (NCC)
- Mutual Information (MI)

- Various registration algorithms uses different metric

Landmark-based Registration

1. Identifying corresponding points in the images and
2. Inferring the image transformation from the points
- Extrinsic landmarks
  - Artificial/foreign objects attached to the patient
- Intrinsic landmarks
  - Internal anatomical or image structures
### Landmark-based Algorithms

<table>
<thead>
<tr>
<th>When correspondences are known</th>
<th>When correspondences are unknown</th>
</tr>
</thead>
<tbody>
<tr>
<td>Singular Value Decomposition (SVD: Shönemann, Farrell, 1966)</td>
<td>Iterative closest point (ICP)</td>
</tr>
<tr>
<td></td>
<td>Procrustean optimum</td>
</tr>
<tr>
<td></td>
<td>Quasi-exhaustive searches, graph matching and dynamic programming approaches</td>
</tr>
</tbody>
</table>

### Application of Extrinsic Landmarks

**Image Guided Surgery:** Just another image registration problem.

One view is an image....

...and the other is the patient.
Extrinsic Landmarks: Pros & Cons

- **Advantage**
  - registration is **accurate**
  - registration is easy, **fast**, and can be automated.
  - no need for complex optimization algorithms.

- **Disadvantage**
  - Often **invasive** character of the marker objects.
  - Non-invasive markers can be used, but less accurate.
  - Prospective character must be made in the pre-acquisition phase.
Intrinsic Landmarks

• Manual selection
  – Using GUI
  – Sparse set of anatomical landmarks
  – Accuracy will depend on user interaction…

• Automatic selection
  – Image feature selection
  – Corner detector (Harris)
  – Saliency feature detector (MSER)
  – Scale invariant feature detector (SIFT)
  – Not easy to find correspondences…

Metric 1: Fiducial Registration Error

• Fiducial Registration Error
  – Correspondences to be known
  – Given N correspondences: \( y \) in fixed, \( x \) in moving image

\[
FRE(\theta) = \sum_{i=1}^{N} \| y_i - T(x_i | \theta) \|^2
= \sum_{i=1}^{N} \| y_i - (Rx_i + T) \|^2
\]

– Aka: Least-Squares
– Registration Error = squared difference between ground-truth \( y \) and predicted landmark \( T(x) \) locations
– Summed over all landmark points
Minimization of FRE

• Problem:
  – Given two corresponding 3D point sets \( \{x_i\} \) and \( \{y_i\} \)
  – Find the optimal rotation \( R \) and translation \( T \)
  – for rigid transform \( y_i = R x_i + T \)
  – that minimizes FRE

• Simple and fast solution by a matrix algorithm called Singular Value Decomposition (SVD)
  – No intensity used
  – Suppose that correspondences are known

• Three steps
  1. Compute centroid/average and subtract from all points in F and M
  2. Find best rotation around the center by SVD
  3. Find best translation by using the result of 2

Algo: Minimization of FRE

Shoenemann, Farrell, 1966
• Minimizes a positive distance function
• Assumes points have been localized
• Guaranteed to converge
• Does not require a good starting pose
• Always finds global optimum (best soln)
• Can be used for surgical guidance
Step 1

- Center the points

\[
\bar{x} = \frac{\sum x_i}{N} \quad \bar{y} = \frac{\sum y_i}{N}
\]

"Centered" points:

\[
\tilde{x}_i = x_i - \bar{x} \quad \tilde{y}_i = y_i - \bar{y}
\]

Step 2

- Determine rotation \( R \)

\[
H = \sum \tilde{x}_i \tilde{y}_i^T
\]

SVD: \( H = UDV^T \)
where

\[
U^T U = V^T V = I
\]

\[
D = \text{diag}(\lambda_1, \lambda_2, \lambda_3)
\]

\[
\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq 0
\]

\[
R = VD U^T
\]
**Singular Value Decomposition**

\[ H = \sum_{i=1}^{N} \tilde{x}_i \tilde{y}_i^T \]  
Outer product of centered point pairs

- \( H \) can be *decomposed* to a product of matrices
  \[ H = U D V^T \]
- Where \( U \) and \( V \) are orthonormal and \( D \) is 3x3 diagonal matrix

**ANS:** \( R = V D U^T \)

---

**Step 3**

- Determine translation
  \[ t \equiv \bar{y} - R \bar{x} \]

**Before rotation**

<table>
<thead>
<tr>
<th>( \bar{x} )</th>
<th>( \bar{y} )</th>
</tr>
</thead>
</table>

**After rotation, but before translation**

<table>
<thead>
<tr>
<th>( R \bar{x} )</th>
<th>( t )</th>
<th>( \bar{y}^* )</th>
</tr>
</thead>
</table>
Surface-based Registration

• Method
  1. First extracting corresponding surfaces
  2. Then compute the transformation by minimizing some measure of distance between the two surfaces

• Algorithms used
  – The Iterative Closest Point (ICP) Algorithm
  – The “Head and Hat” Algorithm
  – Chamfer matching
    – Alignment of binary structures by means of distance transform

• Main issue
  – Registration accuracy is limited by segmentation accuracy

Algo: Iterative Closest Point Method

Besl and McKay 1992
• Minimizes a positive distance function
• Assumes surfaces have been delineated
• Guaranteed to converge
• Requires a good starting pose
• May not find global optimum (Best soln)
• Can be used for surgical guidance
Start with two surfaces

Reorient one (somehow)
Reorient one (somehow)

Reorient one (somehow)
Pick points on moving surface
Remove moving surface

Points become proxy for surface
Find closest points on stationary surface

Measure the total distance
Remove stationary surface

Points become proxy for surface
Restore stationary surface

Find (new) closest points
Find (new) closest points

Remove stationary surface
Remove stationary surface

Register Points
Register Points, and so on…

Iterative Closest-Point Algorithm:

- Find closest points
- Measure total distance
- Register points

Stop when distance change is small.
ICP is popular

- The popularity of ICP can be accredited to its versatility
- It can be used for
  - point sets,
  - Implicitly/explicitly defined curves,
  - surfaces and
  - volumes
- Computational speed
- Ease of implementation

Needs Segmentation!

- ICP requires surface delineation, which is a problem on its own. (Segmentation needed!)

Example:
Level Set Segmentation
(Dawant et al.)

http://www.vuse.vanderbilt.edu/~dawant/levelset_examples/
Summary

• **Medical Image Registration I**
  • Overview of Medical Image Registration
  • Rigid Registrations
  • Landmark-based: Minimizing Fiducial Registration Error
  • Surface-based: Iterative Closest Points

• **Next Week:**
  – Medical Image Registration II
  – Intensity-based: Mutual Information Maximization
  – Non-rigid Registration
  – B-spline
  – Demons
  – ITK issues