## Project!

- How is your project coming?
- It is a part of a semester long project to change initial plans. That is okay but you want to do it carefully. Consult the instructor.
- Any issues? Consult the TA and instructor before too late.

### Medical Image Segmentation II: Level-Set Performance Evaluation

CSC621-821
Biomedical Imaging and Analysis
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Overview

• Last lecture: Similarity-based
  – Medical Image Segmentation I
  – Thresholding
  – Region Growing
  – Watersheds
  – Classification

• Today’s lecture: Discontinuity-based
  – Medical Image Segmentation II
  – Level-set method
  – Performance Evaluation

Boundary-Based Segmentation

• Delineate a boundary of objects in an image
• Edge Detection (Canny)
  – sensitive to noises
• Morphology (Boundary)
  – also sensitive to noises, binary image
• Region Growing
  – boundary is implicitly dealt with, ambiguous when two regions are adjacent to each other
• Watersheds (On gradient image)
  – cannot delineate boundary of uniform regions
Boundary?

• What is boundary?
  – A set of pixels
  – A set of connected/chained pixels (path!!!)
  – Parametric curves
  – Closed or not closed?
  – Topology (donuts, one cookie vs two cookies)

Deformable Models

• Fit a curve to an image boundary based on its shape and image values until it stabilizes
  1. You first put the initial shape by some way
  2. Then make it iteratively move to fit to the object boundary according to
    – Internal forces (curve/surface properties, Prior/Regularization)
      E.g.: Curvature to keep the object smooth
    – External forces (image properties)
      E.g.: To track the object to the boundary

• Introduce prior knowledge of shape of region boundary!
2D Snakes Example

• 2D Snakes, Kass, Witkin and Terzopoulos 1987

Various Names

• Various names for the same:
  • 2D snakes,
  • Active contours
  • Deformable contours

• 3D balloons
  • Active surfaces
  • Deformable surfaces
Shape evolution as dynamic system

- You put a shape model on an image then let it evolve to make it fit to the image’s contour!

- This can be seen as a dynamic system where the state = shape depends on time (time series)
  - Shape: $S = \{(x_i, y_i) | i = 1, \ldots, N\}$
  - Shape model $M = (S, t)$
  - Initial state: $t=0$
  - Evolution = deriving time series by iterating time from 0 to …. (moving front)

- This allows physics equation $\rightarrow$ Diff. Eq. & Euler method

Two Types

- Two main groups
  - Parametric deformable models (polygon)
  - Geometric deformable models (continuous)

- Parametric
  - Stored as a set of vertices
  - Each vertex moves iteratively

- Geometric
  - Stored as a set of coefficients
  - Sample points first
  - Move each points
  - Then re-interpolate a curve

$y = ax^2 + bx + c = \{a, b, c\}$
Level Set Method

- A type of Geometric Deformable Model
- Issues for snakes
  - Hard to deal with changes in topology
- Level set method
  - Makes it possible to handle such topology change
- Concept for 2D segmentation
  - Define a signed distance function in 3D = 2D + 1D space $Z = \psi(x,y,t)$
  - Contour is defined by a cross section of a distance function at $Z=0$
    - $Z=0$ along the contour, $Z > 0$ outside, $Z < 0$ inside the contour
    - The level set $c_0$ at time $t$ of a distance function $\psi(x,y,t)$
      is the set of pixels $\{ (x,y) \mid \psi(x,y,t) = c_0 \} \subset \text{Level set}$
  - Idea: define $\psi(x,y,t)$ so that the front $\gamma(t)$ corresponds to the contour,
    $\gamma(t) = \{ (x,y) \mid \psi(x,y,t) = 0 \}$ (zero level set/front)

Level Set Function

Zero level set/Front: $\psi(x,y)=0$
Signed Distance Function

Signed distance function to the front $\gamma(t=0)$

$$\psi(x,y,0) = \begin{cases} 
- d(x,y, \gamma) & \text{if } (x,y) \text{ inside the front} \\
0 & \text{on } \gamma \\
d(x,y, \gamma) & \text{outside} 
\end{cases}$$

$$\psi(x,y,t) = \psi(x,y,t) + \Delta \psi(x,y,t)$$

Level Set Evolution

- no movement, only change of values
- the front may change its topology
- the front location may be between samples
Level Set Segmentation

Segmentation with LS:
- Initialize the front $\gamma(0)$
- Compute $\psi(x,y,0)$
- Iterate:
  $$\psi(x,y,t+1) = \psi(x,y,t) + \Delta \psi(x,y,t)$$
  until convergence
- Mark the front $\gamma(t_{end})$

Topology Change

- The zero level set (in blue) at one point in time as a cross-section of the level set surface (in red) with $z=0$
Topology Change, Break

- Later in time the level set surface (red) has moved and the new zero level set (blue) defines the new contour

![Level Set Diagram]

Later in Time: Red Level Set Surface has moved, yielding new Blue Interface

How to Move Level Set Function?

- It is a solution of Hamilton-Jacobi equation for a level set function $\Phi$.
- Recall, anisotropic diffusion! Solutions for differential equations are given iterative fashion

$$\frac{\partial \Phi}{\partial t} + F \cdot \nabla \Phi = 0$$

$$\Phi(t+\Delta t) = \Phi(t) - F \cdot \nabla \Phi \Delta t$$

$$\frac{\partial \Phi}{\partial t} + F \left( \left( \frac{\partial \Phi}{\partial x} \right)^2 + \left( \frac{\partial \Phi}{\partial y} \right)^2 \right)^{1/2} = 0$$

$$\frac{\partial \Phi}{\partial t} \sim \frac{\Phi(t+1) - \Phi(t)}{\Delta t} = -F \nabla \Phi$$

$$\Phi(t + 1) - \Phi(t) = \Delta \Phi = -F \nabla \Phi \Delta t$$
Derivation

- Constraint: level set value $\phi$ of a point on the contour with motion $x(t)$ must always be 0
  $$\phi(x(t), t) = 0$$
- By the chain rule, differentiate $\phi$ over $t$ yields
  $$\frac{d\phi}{dt} + \nabla \phi(x(t), t) \cdot x'(t) = 0$$
- Since $F$ supplies the speed in the outward normal direction
  $$x'(t) \cdot n = F$$
- Hence evolution equation for $\phi$ is
  $$\frac{d\phi}{dt} + F |\nabla \phi| = 0$$

Level Set Boundary Segmentation

- Malladi-Sethian-Vemuri '94
  - $\frac{\partial \psi}{\partial t} = k_I \cdot (F_A + F_G(\kappa)) \cdot |\nabla \psi| = 0$
  - constant "force" (balloon pressure)
  - smoothing "force" depending on the local curvature $\kappa$
  - regularization
  - extension of the speed function $k_I$
  - product of influences
  - link between spatial and temporal derivatives!
  - $\kappa = \text{div} \left( \frac{\nabla \psi}{|\nabla \psi|} \right)$
  - spatial derivative of $\psi$
**Speed Function**

- Speed function:
  - $k_I$ is meant to stop the front propagation at the object’s boundaries
  - When image gradient magnitude is high, slow the propagation
  - Otherwise, make propagation goes faster

\[
k_I(x, y) = \frac{1}{1 + |\nabla G_\sigma * I(x, y)|}
\]

**Implementation Issue**

- It is slow, very high time complexity.
- You must update the function value for all the data point although you are only interested in motions of points in the zero level set (at the front)
Speed Enhancement 1

- Narrow band
  - Chop:93, Adalsteinsson-Sethian:95
  - Only update points whose distance values are near zero
  - Evaluate only a few level sets near the front

```
Outward Band
\Phi(x) = +d

Front Position
\Phi(x) = 0

Inward Band
\Phi(x) = -d
```

Speed Enhancement 2

- Fast Marching Algorithm
  - Tsitsiklis:93,Sethian:95
  - Greedy algorithm to compute the front propagation very fast when assuming that the contour curves only expand or shrink
  - It converts the problem to a stationary formulation on a discrete grid where the contour is guaranteed to cross each grid point at most once
  - T(x,y) is the time it takes for the front to reach (x,y)
  - This must satisfy that F*|\nabla T| = 1
  - A solution of this stationary equation is outward expansion of contour with points with smallest T value
Level Set Method Overview

• Mathematically sounds formulation on boundary-based segmentation
• Fast implementation available by allowing some compromise
• Can handle topological changes (active contours cannot)
• Issues
  – Open curves?
  – Sensitivity to initialization?

Performance Evaluation

• Validation
• Ground-truth/Gold Standard
• Expert Manual Labeling
• Phantom
• Figure of Merit
**Validation**

- Validation experiments are necessary to quantify the performance of a segmentation method.
- This is typically performed by comparing **automated segmentation results with true answers**, called **ground-truth/gold-standard**
- Two types of models for truth:
  - Manually obtained segmentations.
  - The use of physical or computational phantoms.

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**Expert’s Manual Segmentation**

- Trained MDs/Radiologists/Biologists are considered to be experts on biomedical anatomy and pathology
- Using a manual GUI tool to have them segment the region of interests and store them as the ground-truth
- Give **clinical significance** to the experimental results by referring to the current medical knowledge
Expert’s Manual Segmentation

ITK Snap

- A part of ITK, free app to annotate and store information like ground truth segmentation results

Phantom

- Manually obtained segmentations do not guarantee perfect truth model because of inherent human operator flaws (different MDs will give different ans.).
- **Physical** phantoms is an artificial model with known geometry.
- It provides accurate depiction of image acquisition process but typically do not present a realistic representation of anatomy.
- **Computational** phantoms represent anatomy realistically, but usually simulate the image acquisition process by using simplified models.
Figure of Merit

- Once a truth model is available, a figure of merit must be defined for quantifying accuracy and precision of segmentation.

- Typical definitions include:
  
  - **Region information**, such as number of pixels misclassified, ratio of overlapped and non-overlapped pixels between the answer and the truth
    - Dice coeff, Jaccard Index, Sensitivity, Specificity, F-measure
  
  - **Boundary information**, such as average of shortest distances to the true boundary.
    - Euclidean distance etc

Categorization of Segmentation

- **Ground Truth**
  - X1: set of all foreground pixels
  - X2: set of all background pixels

- **Your segmentation result**
  - Y1: set of all pixels segmented as foreground
  - Y2: set of all pixels segmented as background
Categorization of Segmentation

• Classification of results
  – TP: number of foreground pixels correctly segmented as foreground
  – TN: number of background pixels correctly segmented as background
  – FP: number of background pixels segmented as foreground
  – FN: number of foreground pixels segmented as background

Visualization

[Diagram showing a Venn diagram comparing ground truth with results, with TP, TN, FP, FN highlighted]
Dice Coefficient

• Aka: F1 score, F measure and DSC: Dice similarity coefficient
• Similarity measure over sets \((0 \leq DSC \leq 1)\)
  – \(DSC = 1\) when \(X1 = Y1\)
  – \(DSC = 0\) when no intersection between \(X1\) and \(Y1\)

\[
DSC = \frac{2|X1 \cap Y1|}{|X1| + |Y1|}
\]

\[
DSC = \frac{2TP}{2TP + FP + FN}
\]

Jaccard Index

• Another set similarity measure
• Aka: Tanimoto Coefficient

\[
J(X1, Y1) = \frac{|X1 \cap Y1|}{|X1 \cup Y1|} = \frac{|X1 \cap Y1|}{|X1| + |Y1| - |X1 \cap Y1|}
\]

\[
J(X1, Y1) = \frac{TP}{TP + FP + FN}
\]
Jaccard Index

- It is bounded between 0 and 1: $0 \leq J \leq 1$
- It is similarity: similar sets give higher value
- Distance function: similar sets give lower value!

$$d_j(X1, Y1) = 1 - J(X1, Y1)$$
$$= \frac{|X1 \cup Y1| - |X1 \cap Y1|}{|X1 \cup Y1|}$$

Relationship: Dice and Jaccard

- They are explicitly related so you do not need to compute both!
- **Jaccard from Dice**

$$J = \frac{D}{2 - D}$$

- **Dice from Jaccard**

$$D = \frac{2J}{1 + J}$$
Sensitivity

- Aka: True Positive Rate, Recall
- General classification performance measure

\[
\text{Sensitivity} = \frac{TP}{|X_1|} = \frac{TP}{TP + FN}
\]

Specificity

- Aka: True Negative Rate
- General classification performance measure

\[
\text{Specificity} = \frac{TN}{|X_2|} = \frac{TN}{TN + FP}
\]
### Precision

- Aka: Positive Predictive Value
- General classification performance measure

\[
\text{Precision} = \frac{TP}{|Y_1|} = \frac{TP}{TP + FP}
\]

### Accuracy

- Aka: ACC
- General classification performance measure

\[
\text{ACC} = \frac{TP + TN}{|X_1| + |X_2|} = \frac{TP + TN}{TP + FN + TN + FP}
\]
F measure = F1 score = DSC

• Harmonic mean of Precision and Recall

\[
F = 2 \times \frac{\text{Precision} \times \text{Recall}}{\text{Precision} + \text{Recall}} = \frac{2TP}{2TP + FP + FN}
\]

How about contours?

• If closed, derive regions then find Y1 and Y2 then apply above methods.
• But what if ground truth is also given by a set of points along the true boundary contours?
• You need to compare two contours given as two sets of points.
• How? → not trivial (need registration!)
Two step solutions

1) Align two contour shapes so as to find corresponding points from each contour.
2) Then compute average Euclidean distances of corresponding points.
3) If correspondence not known, do (after alignment)
   I. Pick each contour point from contour A then
   II. Find closest point from all points in contour B
   III. Repeat this for all points in A then compute average.

How to align shapes?

1) Procrustes analysis
   • Superimposing two shapes by optimally translating, rotating and scaling
   • Do not find you correspondences
2) Dynamic Time Warping
   • Measuring similarity of time series with different speed
   • Applied to shape matching
   • Find you correspondences
3) Registration in general!
Summary

- **Medical Image Segmentation II**
  - Level-set method
  - Performance Evaluation

- **Make sure your team project is going smooth!**

- **Next Week:**
  - Medical Image Registration I
  - Overview of Registration
  - Rigid Registrations
  - Landmark-based: Minimizing Fiducial Registration Error
  - Surface-based: Iterative Closest Points
  - Intensity-based: Mutual Information Maximization
Summary

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• **Senior Oral Requirement**
  – Submit your entry to iLearn & submit the filled-in papers asap

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Senior Oral Presentation

• Your project presentation can be counted and evaluated for this requirement as long as you participate it significantly.

• Submit your entry to iLearn forum

• Bring a hardcopy of requirement forms. Fill in your and course information asap.