Homework Exercise

- Start project coding work according to the project plan
- Adjust project plans according to my comments (reply iLearn threads)
- **New Exercise**: Install VTK & FLTK. Find a simple hello world apps in VTK & FLTK. Then build/execute them. Send me snapshot of results by iLearn within one week for extra credits.
- Once you finish this. Work on connecting your ITK apps and VTK/FLTK. And incorporate it into your project work.
Overview

• Last lecture
  – Practical Foundation of Digital Image Processing I
    – Spatial Domain Analysis
    – Image Enhancement
    – Point Processing: Intensity Transformations
    – Neighbor Processing: Spatial Smoothing Filtering

• Today’s lecture
  – Practical Foundation of Digital Image Processing II
    – Sharpening Filtering in Spatial Domain

Review: Purpose of Image Filtering

• The basic procedure for image processing for
  – Improving image quality for human perception
  – Extracting information for autonomous machine perception

• Manipulate images for
  – Smoothing, Sharpening, Denoising, Restoration, Compression, Edge detection, Shape morphology

• Transform an image to another image
Review: Spatial Image Filtering: Process

The above is repeated for every pixel in the original image to generate the filtered image.

Review: Spatial Image Filtering: Formula

Filtering can be given in equation form as shown above.
Notations are based on the image shown to the left.
Image Enhancement by Sharpening

- Smoothing filters are used to remove fine details from the original images
- **Sharpening spatial filters** seek to highlight fine detail
  - Remove blurring from images
  - Highlight edges
- Sharpening filters are based on *spatial differentiation*

Spatial Differentiation

- Finding the **derivative** of a function with respect to the spatial variable
  - How function’s output change as location changes
  - Rate of change of a function
  - Slope of a function

\[
f'(a) = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h}
\]
Order of Derivatives

- First derivative
  - Speed: change of location
  - Slope
  - Gradient
  - Divergence

- Second derivative
  - Acceleration: change of speed
  - Curvature
  - Hessian
  - Laplacian

Sharpening Filtering: Examples

- Let’s consider a simple 1 dimensional example
Spatial Differentiation: 1st Derivative

- The discrete approximated formula for the 1st derivative of a function \( f \) is as follows:
  \[
  \frac{\partial f}{\partial x} = f\left(x + 1\right) - f\left(x\right)
  \]

- It’s just the difference between subsequent values and measures the rate of change of the function
- High value at the location of changes

Example: 1st Derivative
Spatial Differentiation: 2\textsuperscript{nd} Derivative

- The discrete approximated formula for the 2\textsuperscript{nd} derivative of a function is as follows:
  \[
  \frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x)
  \]
- Takes into account the values both before and after the current value
- High value at the location of high curvature

Example: 2\textsuperscript{nd} Derivative

- The image strip and its second derivative are shown in the diagram.
Derivative Image Filtering

• Given a N-D image (N = 2, 3…)
• Construct a $k$ by $k$ filter derived from the spatial derivatives
  – $k$ value? 3 or 5 or 4 size of filter?
  – Dimension (N): k by k by k by….
  – Derivative order? First or Second?
  – Continuous to discrete?
• Perform filtering of the input image
  – Convolution
  – How to solve border problem?

1st vs 2nd Derivatives for Enhancement

• The 2nd derivative is more useful for image enhancement than the 1st derivative
  – Stronger response to fine detail
  – Simpler implementation
  – We will come back to the 1st order derivative later on

• The first sharpening filter we will look at is the Laplace filter
  – Based on 2nd spatial derivative
  – Look at a discrete implementation
Laplace Filter Concept

- One of the simplest sharpening filters
- Based on 2\textsuperscript{nd} spatial derivative
- Isotropic
  - Radially symmetric \(\rightarrow\) Concentric Circle
  - Respond equality to any direction
- Sum to zero
- Useful for sharpening and edge detection
  - Laplace sharpening
  - Zero crossing edge detection (later)
- We will look at a discrete implementation

Laplace Filter Derivation

- The 2D Laplacian function is defined by:
  \[ \Delta f = \nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} \]
- where the partial 2\textsuperscript{nd} order derivative in the \(x\) direction is defined as follows:
  \[ \frac{\partial^2 f}{\partial x^2} = f(x+1, y) + f(x-1, y) - 2f(x, y) \]
- and in the \(y\) direction as follows:
  \[ \frac{\partial^2 f}{\partial y^2} = f(x, y+1) + f(x, y-1) - 2f(x, y) \]
Laplace Filter Cond.

• So, the 2D Laplacian can be given as follows:
\[
\nabla^2 f = [f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1)] - 4f(x, y)
\]

• We can easily build a filter based on this:

<table>
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<th></th>
<th>0</th>
<th>1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
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<td>-4</td>
<td>1</td>
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<td>0</td>
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Digital Laplace Filtering

• Using the Laplace filter:

<table>
<thead>
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• Perform the neighborhood transformation via image filtering
Laplace Filter Examples

- Applying the Laplacian to an image we get a new image that highlights edges and other discontinuities

Examples of Laplace Filtering

- Retina
Issues for Laplace Filtering

- The result of a Laplace filtering is not an enhanced image (It indicates Curvature)
- We have to do more work in order to sharpen an image
- **Subtract the Laplacian result from the original image to generate our final sharpened enhanced image**

\[ g(x, y) = f(x, y) - \nabla^2 f \]

Image Enhancement by Laplace Filtering

In the final sharpened image edges and fine detail are much more obvious
Laplace Enhancement

Integrating Steps into Single Filter

- The entire enhancement can be combined into a single filtering operation

\[
g(x, y) = f(x, y) - \nabla^2 f
\]

\[
= f(x, y) - [f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1) - 4f(x, y)]
\]

\[
= 5f(x, y) - f(x+1, y) - f(x-1, y) - f(x, y+1) - f(x, y-1)
\]
Digital Laplace Sharpening

- This gives us a new filter which does the whole job for us in one step

![Example of Laplace Sharpening 1](image)

![Example of Laplace Sharpening 1](image)
Variants (Due to Discretization)

- There are lots of slightly different versions of the Laplacian that can be used:

```
0 1 0
1 -4 1
0 1 0
```
Simple Laplacian

```
1 1 1
1 -8 1
1 1 1
```
Variant of Laplacian
Summary

• Practical Foundation of Digital Image Processing II
  – Sharpening Filtering in Spatial Domain

• Next Week:
  – Practical Foundation of Digital Image Processing III
  – Sharpening Filtering in Spatial Domain cond
  – Edge Detection in Spatial Domain
  – Filter Combination
  – Multiple-Image Operation
  – Frequency Domain Techniques

• Homework Exercise/Project:
  – VTK, FLTK \( \rightarrow \) Extra credit by submitting hello world!
  – Start project coding work according to the project plan
  – Adjust project plans according to my comments (reply iLearn threads)