<table>
<thead>
<tr>
<th>Homework Exercise</th>
</tr>
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<td>• Start project coding work according to the project plan</td>
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<tr>
<td>• <strong>New Exercise:</strong> Install VTK &amp; FLTK. Find a simple hello world apps in VTK &amp; FLTK. Then build/execute them. <strong>Send me snapshot of results by iLearn within one week for extra credits.</strong></td>
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<td>• Once you finish this. Work on connecting your ITK apps and VTK/FLTK. And incorporate it into your project work.</td>
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**Digital Image Processing:**
**Sharpening Filtering in Spatial Domain**

CSC621-821
Biomedical Imaging and Analysis
Dr. Kazunori Okada

Adopted from slides for DT228/4 Digital Image Processing by Dr. Brian Mac Namee at Dublin Institute of Technology
http://www.comp.dit.ie/bmacnamee/gaip.htm
Overview

• Last lecture
  – Practical Foundation of Digital Image Processing I
    – Spatial Domain Analysis
    – Image Enhancement
    – Point Processing: Intensity Transformations
    – Neighbor Processing: Spatial Smoothing Filtering

• Today’s lecture
  – Practical Foundation of Digital Image Processing II
    – Sharpening Filtering in Spatial Domain

Review: Purpose of Image Filtering

• The basic procedure for image processing for
  – Improving image quality for human perception
  – Extracting information for autonomous machine perception

• Manipulate images for
  – Smoothing, Sharpening, Denoising, Restoration, Compression,
    Edge detection, Shape morphology

• Transform an image to another image
Review: Spatial Image Filtering: Process

The above is repeated for every pixel in the original image to generate the filtered image.

Review: Spatial Image Filtering: Formula

Filtering can be given in equation form as shown above.

Notations are based on the image shown to the left.
Image Enhancement by Sharpening

- Smoothing filters are used to remove fine details from the original images
- **Sharpening spatial filters** seek to highlight fine detail
  - Remove blurring from images
  - Highlight edges
- Sharpening filters are based on *spatial differentiation*

Spatial Differentiation

- Finding the **derivative** of a function with respect to the spatial variable
  - How function’s output change as location changes
  - Rate of change of a function
  - Slope of a function

\[
f'(a) = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h}
\]
Order of Derivatives

• First derivative
  – Speed: change of location
  – Slope
  – Gradient
  – Divergence

\[ \nabla I = \frac{\partial I(x,y)}{\partial x} + \frac{\partial I(x,y)}{\partial y} \]

• Second derivative
  – Acceleration: change of speed
  – Curvature
  – Hessian
  – Laplacian

\[ \Delta I = \frac{\partial^2 I(x,y)}{\partial x^2} + \frac{\partial^2 I(x,y)}{\partial y^2} \]

Sharpening Filtering: Examples

• Let’s consider a simple 1 dimensional example
Spatial Differentiation: 1st Derivative

- The discrete approximated formula for the 1st derivative of a function $f$ is as follows:
  \[ \frac{\partial f}{\partial x} = f(x+1) - f(x) \]
  
- It's just the difference between subsequent values and measures the rate of change of the function
- High value at the location of changes

Example: 1st Derivative
Spatial Differentiation: 2\textsuperscript{nd} Derivative

- The discrete approximated formula for the 2\textsuperscript{nd} derivative of a function is as follows:
\[
\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x)
\]
- Takes into account the values both before and after the current value
- High value at the location of high curvature

Example: 2\textsuperscript{nd} Derivative
Derivative Image Filtering

- Given a $N$-D image ($N = 2, 3...$)
- Construct a $k$ by $k$ filter derived from the spatial derivatives
  - $k$ value? $5, 4, 8, 5$
  - Dimension (N): $k$ by $k$ by $k$ by....
  - Derivative order? $1^{st}$ or $2^{nd}$
  - Continuous to discrete?
- Perform filtering of the input image
  - Convolution
  - How to solve border problem?

$1^{st}$ vs $2^{nd}$ Derivatives for Enhancement

- The $2^{nd}$ derivative is more useful for image enhancement than the $1^{st}$ derivative
  - Stronger response to fine detail
  - Simpler implementation
  - We will come back to the $1^{st}$ order derivative later on

- The first sharpening filter we will look at is the Laplace filter
  - Based on $2^{nd}$ spatial derivative
  - Look at a discrete implementation
Laplace Filter Concept

- One of the simplest sharpening filters
- Based on 2\textsuperscript{nd} spatial derivative
- Isotropic
  - Radially symmetric $\rightarrow$ Concentric Circle
  - Respond equality to any direction
- Sum to zero
- Useful for sharpening and edge detection
  - Laplace sharpening
  - Zero crossing edge detection (later)
- We will look at a discrete implementation

Laplace Filter Derivation

- The 2D Laplacian function is defined by:
  $$\Delta f = \nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

- where the partial 2\textsuperscript{nd} order derivative in the $x$ direction is defined as follows:
  $$\frac{\partial^2 f}{\partial x^2} = f(x+1, y) + f(x-1, y) - 2f(x, y)$$

- and in the $y$ direction as follows:
  $$\frac{\partial^2 f}{\partial y^2} = f(x, y+1) + f(x, y-1) - 2f(x, y)$$
Laplace Filter Cond.

- So, the 2D Laplacian can be given as follows:
  \[
  \nabla^2 f = [f(x+1,y) + f(x-1,y)] \nabla f + [f(x,y+1) + f(x,y-1)] \nabla f - 4f(x,y)
  \]

- We can easily build a filter based on this

\[
\begin{array}{ccc}
0 & 1 & 0 \\
1 & -4 & 1 \\
0 & 1 & 0 \\
\end{array}
\]

Digital Laplace Filtering

- Using the Laplace filter

\[
\begin{array}{ccc}
0 & 1 & 0 \\
1 & -4 & 1 \\
0 & 1 & 0 \\
\end{array}
\]

- Perform the neighborhood transformation via image filtering
Laplace Filter Examples

- Applying the Laplacian to an image we get a new image that highlights edges and other discontinuities

Original Image  Laplacian Filtered Image  Laplacian Filtered Image Scaled for Display

Examples of Laplace Filtering

- Retina
### Issues for Laplace Filtering

- The result of a Laplace filtering is not an enhanced image (It indicates Curvature)
- We have to do more work in order to sharpen an image
- Subtract the Laplacian result from the original image to generate our final sharpened enhanced image

\[ g(x, y) = f(x, y) - \nabla^2 f \]

### Image Enhancement by Laplace Filtering

- Original Image
- Laplacian Filtered Image
- Sharpened Image

In the final sharpened image, edges and fine detail are much more obvious.
Laplace Enhancement

Integrating Steps into Single Filter

- The entire enhancement can be combined into a single filtering operation

\[
g(x, y) = f(x, y) - \nabla^2 f
\]

\[
= f(x, y) - [f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1) - 4f(x, y)]
\]

\[
= 5f(x, y) - f(x+1, y) - f(x-1, y) - f(x, y+1) - f(x, y-1)
\]
Digital Laplace Sharpening

- This gives us a new filter which does the whole job for us in one step

Example of Laplace Sharpening 1
Example of Laplace Sharpening 2

Variants (Due to Discretization)

- There are lots of slightly different versions of the Laplacian that can be used:

```
0 1 0
1 -4 1
0 1 0
```

Simple Laplacian

```
1 1 1
1 -8 1
1 1 1
```

Variant of Laplacian

```
-1 -1 -1
-1 9 -1
-1 -1 -1
```
Summary

- **Practical Foundation of Digital Image Processing II**
  - Sharpening Filtering in Spatial Domain

- **Next Week:**
  - **Practical Foundation of Digital Image Processing III**
  - Sharpening Filtering in Spatial Domain cond
  - Edge Detection in Spatial Domain
  - Filter Combination
  - Multiple-Image Operation
  - Frequency Domain Techniques

- **Homework Exercise/Project:**
  - VTK, FLTK → Extra credit by submitting hello world!
  - Start project coding work according to the project plan
  - Adjust project plans according to my comments (reply iLearn threads)