Digital Image Processing: Sharpening Filtering in Spatial Domain
CSC621-821
Biomedical Imaging and Analysis
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Homework Exercise

• Start project coding work according to the project plan
• Adjust project plans according to my comments (update any changes by replying the iLearn threads)
• New Exercise: Install VTK & FLTK. Find a simple hello world apps in VTK & FLTK. Then build/execute them. Submit screenshot images of the results to the iLearn link by the next class meeting time for EC.
• Once you finish this. Work on connecting your ITK apps and VTK/FLTK. And incorporate it into your project work.
Overview

• Last lecture
  – Practical Foundation of Digital Image Processing I
    – Spatial Domain Analysis
    – Image Enhancement
    – Point Processing: Intensity Transformations
    – Neighbor Processing: Spatial Smoothing Filtering

• Today’s lecture
  – Practical Foundation of Digital Image Processing II
    – Sharpening Filtering in Spatial Domain

Review: Purpose of Image Filtering

• The basic procedure for image processing for
  – Improving image quality for human perception
  – Extracting information for autonomous machine perception

• Manipulate images for
  – Smoothing, Sharpening, Denoising, Restoration, Compression,
    Edge detection, Shape morphology

• Transform an image to another image
The above is repeated for every pixel in the original image to generate the filtered image.

Filtering can be given in equation form as shown above. Notations are based on the image shown to the left.
Image Enhancement by Sharpening

- Smoothing filters are used to remove fine details from the original images
- **Sharpening spatial filters** seek to highlight fine detail
  - Remove blurring from images
  - Highlight edges
- Sharpening filters are based on **spatial differentiation**

Spatial Differentiation

- Finding the **derivative** of a function with respect to the spatial variable
  - How function’s output change as location changes
  - Rate of change of a function
  - Slope of a function

\[ f'(a) = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h} \]
Order of Derivatives

• First derivative
  – Speed: change of location
  – Slope
  – Gradient
  – Divergence
    \[ \nabla I = \frac{\partial I(x,y)}{\partial x} + \frac{\partial I(x,y)}{\partial y} \]

• Second derivative
  – Acceleration: change of speed
  – Curvature
  – Hessian
  – Laplacian
    \[ \Delta I = \frac{\partial^2 I(x,y)}{\partial x^2} + \frac{\partial^2 I(x,y)}{\partial y^2} \]

Sharpening Filtering: Examples

• Let’s consider a simple 1 dimensional example

Spatial Differentiation: 1st Derivative

• The discrete approximated formula for the 1st derivative of a function $f$ is as follows:

$$\frac{\partial f}{\partial x} \approx f(x+1) - f(x) \quad \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \quad \uparrow h \approx 1$$

• It’s just the difference between subsequent values and measures the rate of change of the function

• High value at the location of changes

Example: 1st Derivative
Spatial Differentiation: 2\textsuperscript{nd} Derivative

- The discrete approximated formula for the 2\textsuperscript{nd} derivative of a function is as follows:

\[
\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x)
\]

- Takes into account the values both before and after the current value
- High value at the location of high curvature

Example: 2\textsuperscript{nd} Derivative
Derivative Image Filtering

• Given a N-D image (N = 2, 3…)
• Construct a \( k \) by \( k \) filter derived from the spatial derivatives
  – \( k \) value? \( 3, 4, 5, \ldots \)
  – Dimension (N): \( k \) by \( k \) by \( k \) by….
  – Derivative order? \( 1^{st}, 2^{nd} \)
  – Continuous to discrete?

• Perform filtering of the input image
  – Convolution
  – How to solve border problem?

\[ \text{1}\text{st vs 2}\text{nd Derivatives for Enhancement} \]

• The 2^{nd} derivative is more useful for image enhancement than the 1^{st} derivative
  – Stronger response to fine detail
  – Simpler implementation
  – We will come back to the 1^{st} order derivative later on

• The first sharpening filter we will look at is the **Laplace filter**
  – Based on 2^{nd} spatial derivative
  – Look at a discrete implementation
Laplace Filter Concept

- One of the simplest sharpening filters
- Based on 2\textsuperscript{nd} spatial derivative
- Isotropic
  - Radially symmetric $\rightarrow$ Concentric Circle
  - Respond equality to any direction
- Sum to zero $\rightarrow$ Sums to one.
- Useful for sharpening and edge detection
  - Laplace sharpening
  - Zero crossing edge detection (later)
- We will look at a discrete implementation

Laplace Filter Derivation

- The 2D Laplacian function is defined by:
  \[ \Delta f = \nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} \]
  
  where the partial 2\textsuperscript{nd} order derivative in the $x$ direction is defined as follows:
  \[ \frac{\partial^2 f}{\partial x^2} = f(x+1, y) + f(x-1, y) - 2f(x, y) \]
  - and in the $y$ direction as follows:
  \[ \frac{\partial^2 f}{\partial y^2} = f(x, y+1) + f(x, y-1) - 2f(x, y) \]
Laplace Filter Cond.

• So, the 2D Laplacian can be given as follows:
  \[ \nabla^2 f = [f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1)] - 4f(x, y) \]

• We can easily build a filter based on this

```
0 1 0
1 -4 1
0 1 0
```

Digital Laplace Filtering

• Using the Laplace filter

```
0 1 0
1 -4 1
0 1 0
```

• Perform the neighborhood transformation via image filtering
Laplace Filter Examples

- Applying the Laplacian to an image we get a new image that highlights edges and other discontinuities

Examples of Laplace Filtering

- Retina
Issues for Laplace Filtering

- The result of a Laplace filtering is not an enhanced image (It indicates Curvature)
- We have to do more work in order to sharpen an image
- Subtract the Laplacian result from the original image to generate our final sharpened enhanced image

\[ g(x, y) = f(x, y) - \nabla^2 f \]

Image Enhancement by Laplace Filtering

In the final sharpened image, edges and fine detail are much more obvious
Laplace Enhancement

Integrating Steps into Single Filter

- The entire enhancement can be combined into a single filtering operation

\[ g(x, y) = f(x, y) - \nabla^2 f \]

\[ = f(x, y) - \left[ f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1) - 4f(x, y) \right] \]

\[ = 5f(x, y) - f(x+1, y) - f(x-1, y) - f(x, y+1) - f(x, y-1) \]
Digital Laplace Sharpening

- This gives us a new filter which does the whole job for us in one step

Example of Laplace Sharpening 1
Example of Laplace Sharpening 2

Variants (Due to Discretization)

- There are lots of slightly different versions of the Laplacian that can be used:

<table>
<thead>
<tr>
<th>Simple Laplacian</th>
<th>Variant of Laplacian</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 1 0</td>
<td>1 1 1</td>
</tr>
<tr>
<td>1 -4 1</td>
<td>1 -8 1</td>
</tr>
<tr>
<td>0 1 0</td>
<td>1 1 1</td>
</tr>
</tbody>
</table>

-1 -1 -1
-1 9 -1
-1 -1 -1
Summary

- **Practical Foundation of Digital Image Processing II**
  - Sharpening Filtering in Spatial Domain

- **Next Week:**
  - **Practical Foundation of Digital Image Processing III**
  - Sharpening Filtering in Spatial Domain cond
  - Edge Detection in Spatial Domain
  - Filter Combination
  - Multiple-Image Operation
  - Frequency Domain Techniques

- **Homework Exercise/Project:**
  - VTK, FLTK → Extra credit by submitting hello worlds SS to iLearn!
  - Start project coding work according to the project plan
  - Adjust project plans according to my comments (reply iLearn threads)