Midterm #2

- HW5 Due Now & Pickup HW6: due on 11/7
- Midterm #2 in Five Days on 10/29 (Relation/Function)
  - 5 questions on Relations and Functions
    - 3 Relations/Functions, 2 Counting
  - Bring: 1-page handwritten cheat-sheet, calculator, scratch papers
  - No reentry after leaving
  - Study with example problems BOTH in Lec Notes and HWs. For more look into the TextBook
  - MORE DIFFICULT THAN MT1!!
- This Lecture
  - Review for Midterm #2: HWs 4 & 5

Homework 4
(Total 25 pts)

CSC230 Discrete Math
Kazunori Okada
Determine whether the binary relation $R$ on the set of all people is reflexive, symmetric, antisymmetric, transitive where $(a,b) \in R$ if and only if: (i) $a$ is taller than $b$

- ANS
  
  reflexive: No  
  symmetric: No  
  anti-symmetric: Yes  
  transitive: Yes

$\forall (a,b) \in R \rightarrow (b,a) \in R$

$R = R^{-1}$


Determine whether the relation $R$ on the set of all people is reflexive, symmetric, antisymmetric, transitive where $(a,b) \in R$ if and only if: (ii) $a$ has the same last name as $b$

- ANS

  reflexive: Yes  
  symmetric: Yes  
  anti-symmetric: No  
  transitive: Yes
HW#4 Q3 (4pt)

Let R be the relation on the set \{0,1,2,3\} with ordered pairs (0,1),(1,1),(1,2),(2,0),(2,2) and (3,0). Find the (i) reflexive closure of R

• ANS

\[ R = \{(0,1), (1,1), (1,2), (2,0), (2,2), (3,0)\} \]

Reflexive Closure

\[ \text{Id} = \{(0,0), (1,1), (2,2), (3,3)\} \]

\[ R \cup \text{Id} = \{(0,0), (0,1), (1,1), (1,2), (2,0), (2,2), (3,0), (3,3)\} \]

HW#4 Q3 (4pt)

Let R be the relation on the set \{0,1,2,3\} with ordered pairs (0,1),(1,1),(1,2),(2,0),(2,2) and (3,0). Find the (ii) symmetric closure of R

• ANS

\[ R^S = \{(0,1), (1,1), (1,2), (2,2), (3,3)\} \]

\[ R \cup R^S = \{(0,1), (1,1), (1,2), (2,2), (3,3)\} \]

Two-Times - Repeat - Universal (SxS)
Let $R$ be the relation on the set of ordered pairs of positive integers such that $((a,b),(c,d)) \in R$ if and only if $ad = bc$. (i) Show that $R$ is an equivalence relation.

**ANS**

- Reflexive: $(a,b),(a,b) \in R \Rightarrow a \cdot b = b \cdot a$
- Symmetric: if $(a,b),(c,d) \in R \Rightarrow ad = bc \Rightarrow cb = da$
- Transitive: $(a,b),(c,d) \in R$ and $(c,d),(e,f) \in R \Rightarrow ((c,d),(e,f)) \in R \Rightarrow ad = bc \Rightarrow af = be$ since $c > 0, d > 0$

$\Rightarrow ((a,b),(e,f)) \in R$.

Let $R$ be the relation on the set of ordered pairs of positive integers such that $((a,b),(c,d)) \in R$ if and only if $ad = bc$. (ii) What is $[(1,2)]$, equivalence class of $(1,2)$?

**ANS**

$\frac{a}{b} = \frac{2}{1} \Rightarrow a = 2b \Rightarrow 2a = b \Rightarrow \frac{b}{a} = 2$.

$2 \cdot a = b \Rightarrow 2 \cdot \text{any pair } (c,d) \text{ with } \frac{d}{c} = 2 \Rightarrow \text{any pair } (c,2c) \Rightarrow c \in \mathbb{N}^+$

Or $\frac{c}{d} \Rightarrow \{ (1,2), (1,4), (3,6), (4,8), (5,10), \ldots \}$

$\{ (c,d) \mid c,d \in \mathbb{Z}^+ \text{ and } d = 2c \}$. 

$S = \{ (a,b) \mid a > 0, b > 0 \text{ and } a \neq b \}$. 

$\forall a > 0, b > 0 \Rightarrow a > b \Rightarrow a \cdot a > b \cdot b \Rightarrow a^2 > b^2 \Rightarrow a > b$. 

Or $\forall a > 0, b > 0 \Rightarrow a = b \Rightarrow a > b$.
HW#5 Q1 (2pt)

Determine whether each of these functions is bijection from $\mathbb{R}$ to $\mathbb{R}$ (explain your solutions) (a) $f(x) = -3x + 4$

- Ans

To show $f(x)$ is bijection, we have to show that $f$ is 1-1 and Onto.

1-1: For all $a, b \in \mathbb{R}$, if $f(a) = f(b)$ then $-3a + 4 = -3b + 4$ thus $3a = 3b$ and hence $a = b$.

Onto: For all $b \in \mathbb{R}$ (codomain), there exists $a$ such that $b = f(a) = -3a + 4$ then $a = \frac{b - 4}{3}$. Since $b \in \mathbb{R}$, $a = \frac{b - 4}{3} \in \mathbb{R}$ (domain).

Thus $f(x)$ is onto.

$\therefore f(x)$ is bijection.
HW#5 Q1 (2pt)

Determine whether each of these functions is bijection from $\mathbb{R}$ to $\mathbb{R}$ (explain your solutions) (b) $f(x) = \frac{x+1}{x+2}$

- ANS
  - Not bijection, not a function
  - $f(-2)$ is undefined
  - $f(x) = 1 = \frac{x+1}{x+2}$
  - $x+1 = x+2$ is not true

HW#5 Q2 (1.5pt)

Let $f(x) = x^2 + 1$, $g(x) = x + 2$ are functions from $\mathbb{R}$ to $\mathbb{R}$.

Find (i) $f \circ g$

- ANS
  - $(a, b)$, $(b, c)$
  - $f(g(x)) = (x+2)^2 + 1 = x^2 + 4x + 5$
  - $(a, c)$

OR

- $f(x) = \{\ldots (-1, 0), (0, 1), (1, 2), (3, 10), (4, 17), \ldots\}$
- $g(x) = \{\ldots (-1, 1), (0, 2), (1, 3), (3, 5), (4, 6), \ldots\}$
HW#5 Q2 (1.5pt)

Let \( f(x) = x^2 + 1 \), \( g(x) = x + 2 \) are functions from \( \mathbb{R} \times \mathbb{R} \).
Find (ii) \( g \circ f \)

- ANS
  \[
  g \circ f = g(f(x)) = (x^2 + 1) + 2 = x^2 + 3
  \]

HW#5 Q2 (1.5pt)

Let \( f(x) = x^2 + 1 \), \( g(x) = x + 2 \) are functions from \( \mathbb{R} \times \mathbb{R} \).
Find (iii) \( f + g \)

- ANS
  \[
  f + g = f(x) + g(x) = x^2 + 1 + x + 2 = x^2 + x + 3
  \]
HW#5 Q2 (1.5pt)

Let $f(x) = x^2 + 1$, $g(x) = x + 2$ are functions from $\mathbb{R} \times \mathbb{R}$.

Find (iv) $fg$

- ANS

$$fg = f(g(x)) = (x^2 + 1)(x + 2) = x^3 + x + 2x^2 + 2 = x^3 + 2x^2 + x + 2$$

HW#5 Q4 (2pt)

Find values of these summations: (i) $\sum_{i=1}^{3} \sum_{j=1}^{2} (i - j)$

- ANS

$$\sum_{i=1}^{3} \sum_{j=1}^{2} (i - j) = \sum_{i=1}^{3} \frac{1}{2} \left[ (i - 1)(i - 2) \right] = \sum_{i=1}^{3} \frac{(i - 1)(i - 2)}{2} = \sum_{i=1}^{3} \frac{2(i - 3)}{2} + \sum_{i=1}^{3} (i - 3) = -1 + 1 + 3 = 3$$
HW#5 Q4 (2pt)

Find values of these summations: (ii) \( \sum_{i=0}^{2} \sum_{j=0}^{3} i^2 j^3 \)

- **ANS**
  \[
  \sum_{i=0}^{2} \sum_{j=0}^{3} i^2 j^3 = \sum_{i=0}^{2} \left( \sum_{j=0}^{3} i^2 j^3 \right) = \sum_{i=0}^{2} (0 + 2^2 1^3 + 2^2 2^3 + 2^2 3^3) = 36 \cdot 5 = 180
  \]

HW#5 Q5 (3pt)

Determine whether each of these sets is countable or uncountable. Explain your solutions. (i) the odd negative integers

- **ANS**
  \[ X = \{ x \in \mathbb{Z} \mid x = 2n+1 \text{ for some } n \in \mathbb{Z} \} \]
  Let \( f: \mathbb{N} \to X \) defined as
  \[
  f(n) = \left\{ \begin{array}{ll}
  0, & \text{if } n = 0 \\
  -1, & \text{if } n = 1 \\
  -3, & \text{if } n = 2 \\
  -5, & \text{if } n = 3 \\
  -7, & \text{if } n = 4 \\
  \end{array} \right.
  \]
  Then \( f(n) \) can be defined as \( f(n) = -(2n+1) \) for \( n \geq 0 \) and \( f(n) \) is one-to-one correspondence. \( f(0) = 0 \) and \( f(1) = -1 \) and \( f(2) = -3 \).
HW#5 Q5 (3pt)

Determine whether each of these sets is countable or uncountable. Explain your solutions. (ii) the real number between 1 and 2

• ANS

• ANS

HW#5 Q8 (2.5pt)

Suppose that there are nine students in a class who are juniors or seniors. (a) Show that the class must have at least five junior students or at least five senior students

• ANS

• ANS
Suppose that there are nine students in a class who are JRs or SRs. (b) Show that the class must have at least three junior students or at least seven senior students.

- **ANS**

Proof by contradiction.

\[ \Rightarrow \text{Class does not have at least } 3 \text{ JR student and at least } 7 \text{ SR student} \]

\[ \Rightarrow \text{Class has at most } 2 \text{ JR and Class has at most } 6 \text{ SR} \]

\[ \Rightarrow \text{Class has at most } 8 \text{ student} \]

\[ \Rightarrow \text{Contradiction} \]

\[ \Rightarrow \]